HYDRAULIC RESISTANCE AND HEAT AND MASS EXCHANGE DURING TURBULENT FLOW OVER SURFACES WITH SLIGHT ROUGHNESS

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Flow over a surface with slight roughness is examined. It is established that at high Prandtl or Schmidt numbers such roughness can markedly increase the coefficients of heat and mass exchange.

The hypothesis has been expressed in [1] that the protrusions of irregularities on a surface with relatively slight roughness exert an additional turbulizing effect on the laminar sublayer.

This can be justified physically as follows. Directly at the surface the liquid moves by passing over the elements of roughness. Consequently the particles of liquid will possess additional vibrational motions compared with a smooth surface. These vibrations of the flow increase the level of turbulization and can lead to the appearance of additional shear stresses. It must be noted that the concept itself of a wall boundary in the presence of roughness is somewhat indeterminant if this boundary is measured with respect to the normal to the direction of motion of the external flow. In this case the wall boundary represents an indistinct range of sizes of the protrusions. However, one can arbitrarily take as the wall boundary the coordinate where the longitudinal component of the velocity is equal to zero. In this form it is possible to allow for the presence of the additional turbulization of the flow by a simple shift in the origin of the coordinates.

If one adopts for a smooth surface the dependence of [2] corresponding to a fourth power law for the penetration of velocity pulsations into the laminar sublayer, the equation for calculating the velocity profile for a rough surface will have the form

$$\varphi = \int_0^\eta \frac{1}{1 + 5.624\,[0.0805\,(\eta + \eta_r)]^4 \Big/ \left\{1 + \dfrac{[0.0805\,(\eta + \eta_r)]^2}{0.2667\sqrt{\eta + \eta_r}}\right\}^2}\, d\eta. \tag{1}$$

At $\eta_r = 0$ Eq. (1) corresponds to the uninterrupted flat velocity profile of a smooth surface [2].

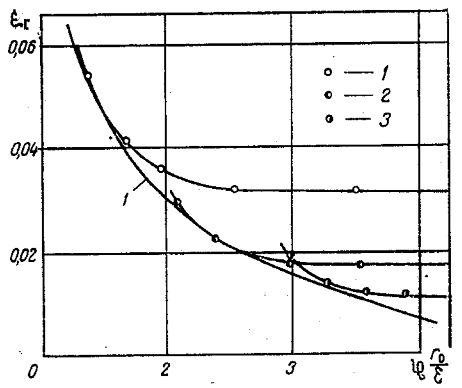

Fig. 1. Calculated coefficients of hydraulic resistance from Eqs. (6) and (7) for slight roughness. Envelope curve is calculated from Eq. (8): 1) Re = 10^4; 2) 10^5; 3) 10^6.

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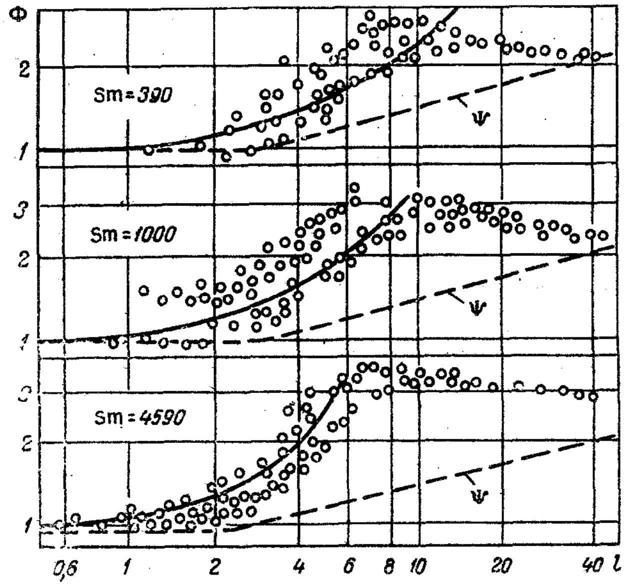

Fig. 2. Comparison of experiments [10] and calculations from Eq. (14). Solid curves: calculations; dashed curves: experiments on hydraulic resistance.

The operation of the numerical calculation of the integral (1) is accomplished very simply on an electronic computer by the same program as for a smooth surface, taking the initial values

$$\eta = \eta_r; \qquad \varphi = 0.$$

Then calculating from the profile the average velocity for the motion of a liquid in a round tube

$$\varphi_f = \frac{2}{\eta_0^2} \int\limits_0^{\eta_0} \varphi(\eta_0 - \eta)\, d\eta, \tag{2}$$

one can calculate the coefficient of resistance

$$\xi = 8/\varphi_f^2, \tag{3}$$

corresponding to the Reynolds number

$$\mathrm{Re}_f = 2\varphi_f \eta_0 \tag{4}$$

and the relative roughness

$$\frac{y_r}{r_0} = \frac{\eta_r}{\eta_0}. \tag{5}$$

Here we use the well-known equations (2)-(5) which follow from calculations in universal coordinates.

As a result of the calculations numerical values were obtained for the coefficients of resistance, which are correlated with an accuracy of ±2% by the dependence

$$\Psi = \frac{\xi}{\xi_s} = 1 \Big/ \left(1 - 1.1 \cdot 10^{-2} \mathrm{Re}_f^{0.875} \frac{y_r}{r_0}\right). \tag{6}$$

The structure of this equation and of the others presented below is obtained by the method of relative correspondence [3] on a simple two-layer model. This method, which has not yet received due appreciation, provides the basis for the application of rougher calculating models in the case when the results of the calculation are represented in relative values. A linear dependence between the displacement y_r of the origin of coordinates and the height δ of the roughness is assumed in [1]

$$y_r = c\delta. \tag{7}$$

By comparison with experiments on the hydraulic resistance in tubes with a sandy surface roughness it is found that c = 0.127.

A curve constructed from the Nikuradse equation

$$\xi_{\mathrm{r}} = \left(0.88 \ln \frac{r_0}{\delta} + 1.65\right)^{-2}, \tag{8}$$

which determines the coefficient of resistance in the region of a self-similar mode, and curves constructed from Eq. (6) with allowance for (7) are presented in Fig. 1. As is seen, with only the help of the single experimental constant 0.127 it is possible to correlate the tangency of the curves of slight roughness with the curve of (8). If instead of one constant each curve corresponded to its own value or the dependence (7) had a more complicated form the validity of J. C. Rotta's hypothesis would be in doubt. In the present case one can assert that this hypothesis is confirmed.

The equation

$$y_{\mathrm{r}} = 0.5\delta, \tag{9}$$

which also correlates all the curves of coefficients of hydraulic resistance in the region of slight roughness, is obtained analogously for technical roughness determined by the Kolburn graph [4].

The dependence (6) is valid only for relatively slight roughness, when the protrusions of the irregularities lie within the limits of the laminar sublayer or partly within the transitional layer. This zone is approximately limited by the value

$$\Psi = 1 - 1.15. \tag{10}$$

It is interesting to bring in the results of a calculation of heat and mass exchange in the presence of the effect of slight roughness. Here the calculating equation in dimensionless universal variables has the form

$$\Theta = \int\limits_0^{\eta} \frac{1}{\dfrac{1}{\mathrm{Pr}} + \varepsilon \dfrac{5.624\,[0.0805\,(\eta + \eta_{\mathrm{r}})]^4}{\left\{1 + \dfrac{[0.0805\,(\eta + \eta_{\mathrm{r}})]^2}{0.2667\sqrt{\eta + \eta_{\mathrm{r}}}}\right.}} \, d\eta. \tag{11}$$

Here ε is the coefficient of dissimilarity in the dispersion of heat content and momentum, taken as equal to 1.1. The integral of (11) was calculated on an electronic computer like (1). The mean volumetric temperature for the conditions of motion of a liquid in a round tube was determined from the temperature and velocity profiles and the coefficient of convective heat exchange was calculated. The results of the calculations are correlated with an accuracy of ±5% in relative form by the equation

$$\Phi = \frac{\alpha}{\alpha_0} = 1/1 - 1.1 \cdot 10^{-2}\,\mathrm{Re}_f^{0.875}\,\mathrm{Pr}^{0.25}\,\frac{y_{\mathrm{r}}}{r_0}. \tag{12}$$

It follows from a comparison with (6) that when $\mathrm{Pr} > 1$ the effect of slight roughness on the coefficient of convective heat exchange is more important than the effect on the coefficient of hydraulic resistance. This effect was observed in the experiments of [5, 6], although without an explanation of the causes.

According to [5] the characteristic indicator of the excess of the growth in the coefficient of convective heat exchange compared with the coefficient of hydraulic resistance is the ratio Φ/Ψ, which can be converted into the following form for sandy roughness with allowance for (6) and (12):

$$\frac{\Phi}{\Psi} = \frac{1 - 1.39 \cdot 10^{-2}\, l}{1 - 1.39 \cdot 10^{-2}\mathrm{Pr}^{0.25}\, l}, \tag{13}$$

where

$$l = \mathrm{Re}\sqrt{\frac{\xi}{8}} \cdot \frac{\delta}{d} = \mathrm{Re}^{0.875}\sqrt{\frac{0.3164}{8}} \cdot \frac{1}{2}\,\frac{\delta}{r_0} \simeq 0.1\mathrm{Re}^{0.875}\frac{\delta}{r_0}.$$

A mode with $\Phi/\Psi > 1$ has been observed in experiments on the heat exchange of water at $\mathrm{Pr} = 5$ and $l = 5$–20.

It is now possible to compare the dependence (12) with the experiments of Dawson and Trass [10] on diffusion which were conducted by electrochemical means. Surfaces with artificial roughness in the form of V-shaped channels were used in the experiments. For comparison with the experiment the equation (7) for sandy roughness and the parameter l from (13) were introduced into Eq. (12)

$$\Phi = 1/1 - 1.39 \cdot 10^{-2}\,\mathrm{Pr}^{0.25}\, l. \tag{14}$$

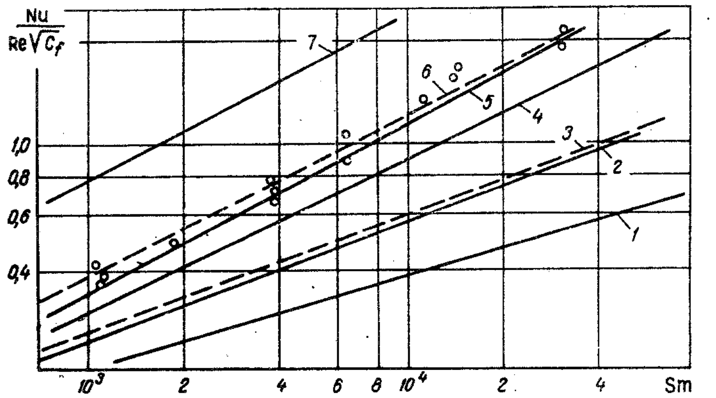

Fig. 3. Comparison of experimental data on diffusion at large Schmidt numbers with calculated values for different "diffusion" roughness: 1) calculation from Eq. (20), conditionally corresponding to a smooth surface (fourth power law for penetration of velocity pulsations); 2) from the equation

$$\mathrm{Nu} = \frac{1}{8 \cdot 4.5} \sqrt{\xi}\, \mathrm{Re}\, \mathrm{Pr}^{1/3}$$

(third power law); 3) experiment of [8], Eq. (21); 6) calculation and experimental points [9], Eq. (22) (second power law); 7) experiments of I. I. Barker and R. E. Treibol; 2, 4, 5) calculation from Eqs. (20), (12), and (9) at r_0/δ = 800, 326, and 213.

The experimental results and their comparison with calculations from this equation are presented in Fig. 2. As is seen, the theoretical curves are confirmed on the average by the experiment, although the relatively large scatter of the experimental points must be noted.

If the condition of hydraulic smoothness is limited by a 2% change in the coefficient of resistance then from (6) and (7) we obtain

$$\frac{r_0}{\delta} \gtrsim 0.07\, \mathrm{Re}^{0.875}. \tag{15}$$

Accordingly the condition of hydraulic smoothness for technical roughness is

$$\frac{r_0}{\delta} \gtrsim 0.27\, \mathrm{Re}^{0.875}. \tag{16}$$

The condition of thermal smoothness for sandy and technical roughnesses will be determined similarly from (13) by the dependences

$$\frac{r_0}{\delta} \gtrsim 0.07\, \mathrm{Re}^{0.875}\, \mathrm{Pr}^{0.25}; \qquad \frac{r_0}{\delta} \gtrsim 0.27\, \mathrm{Re}^{0.875}\, \mathrm{Pr}^{0.25}. \tag{17}$$

A mode of hydraulic smoothness and thermal roughness exists in the interval:

for sandy roughness

$$0.07\, \mathrm{Re}^{0.875}\, \mathrm{Pr}^{0.25} > \frac{r_0}{\delta} > 0.07\, \mathrm{Re}^{0.875}; \tag{18}$$

for technical roughness

$$0.27\, \mathrm{Re}^{0.875}\, \mathrm{Pr}^{0.25} > \frac{r_0}{\delta} > 0.27\, \mathrm{Re}^{0.875}. \tag{19}$$

The presence of this intermediate mode can explain the disagreement in the experimental results of a number of authors on turbulent mass exchange at high Schmidt numbers.

The experiments known at present include: those of V. G. Levich [7] and other authors, which for large Schmidt numbers lead to the equation

$$\mathrm{Nu} = 0.041 \sqrt{\xi}\, \mathrm{Re}\, \mathrm{Sm}^{0.25}; \tag{20}$$

the experiments of P. Harriott and R. Hamilton [8], which are correlated by the equation

$$Nu = 0.0096\, Re^{0.913}\, Sm^{0.346}; \tag{21}$$

the experiments of N. K. Kishinevskii et al. [9] with the corresponding correlating equation

$$Nu = 0.02\, Re^{0.833}\, Sm^{0.5}. \tag{22}$$

In the first case the correlating equation leads to a fourth power law for the penetration of velocity pulsations into the laminar sublayer, in the second case it leads to a third power law, while Eq. (22) corresponds to a second power law. A number of experiments of other authors are also known which either confirm one of the equations (20)(22) or are close to them.

From a practical aspect the difference in the equations presented is inadmissible, since even at Sm = 40,000 the disagreement in the calculations reaches 100% or more.

An analysis of the methods of the experiments shows that only the essential condition of hydraulic smoothness was controlled in the treatment of the results. If the presence of thermal and diffusion roughness is assumed in the experiments, which is quite probable in the presence of dissolving of the surface, the reason for the discrepancies noted becomes clear enough. The results of calculations by Eqs. (20)-(22) and a calculation by Eq. (20) with a correction according to (12) are plotted in Fig. 3. It was assumed, although conditionally, that Eq. (20), which corresponds to a fourth power law, characterizes the mass exchange of a smooth surface or is in a mode of diffusion smoothness in accordance with the condition (17).

By selecting the roughness according to the assumption made in the experiments of [8] and [9] and by the approximate matching in this way of the experimental curves and the theoretical curves from Eqs. (21) and (22) at small Schmidt numbers ($Sm < 10^3$) it was possible to satisfactorily match the curves in the entire range of Schmidt numbers corresponding to the experiments.

In this case the slope of the curve in the coordinates

$$\frac{Nu}{Re\sqrt{\xi}} = f(Sm)$$

increases in proportion to the increase in the absolute value of the coefficient of mass exchange just as occurs in the calculations from the equations (20)-(22).

NOTATION

$\varphi = w/\sqrt{\tau/\rho}$	is the dimensionless velocity;
$\eta = y\sqrt{\tau/\rho}/\nu$	is the dimensionless transverse coordinate;
$\eta_r = y_r\sqrt{\tau/\nu}$	is the dimensionless displacement of coordinates;
y_r	is the displacement of coordinates;
τ	is the shear stress at wall;
ρ	is the density;
w	is the velocity;
φ_f	is the average dimensionless velocity;
η_0	is the dimensionless radius of tube;
ξ	is the coefficient of hydraulic resistance;
Re_f	is the Reynolds number;
r_0	is the radius of tube;
δ_r	is the height of roughness protrusions;
Ψ	is the ratio of coefficients of hydraulic resistance of rough and smooth surfaces;
$\Theta = (T-T_w)\rho c_p\sqrt{\tau/\rho}/q$	is the dimensionless temperature;
Pr	is the Prandtl number;
ε	is the coefficient of dissimilarity of diffusion of heat content and momentum;
Φ	is the ratio of coefficients of heat and mass exchange of rough and smooth surfaces;
Sm	is the Schmidt number.

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